

02-10 Portfolio Optimization and the Efficient Frontier

Compiled by Shipra De, Fall 2016

Overview

Given:
• Set of equities
• Target return

Find:
• Allocation to each equity that minimizes risk



- Suppose you have a set of stocks that you've determined are good investments. How much of your portfolio should you invest in each?
- There are many potential answers to that question. In this lesson we take a look at the approach called mean variance optimization, or portfolio optimization.
- The specific question we're looking to answer is this. Given a set of equities and a target return, find an allocation to each equity that minimizes risk.

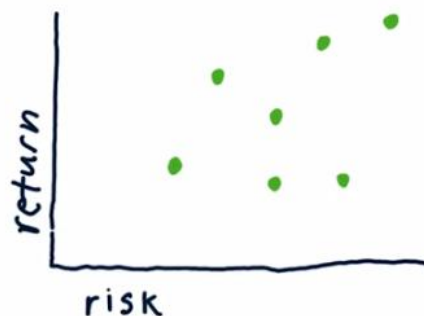
What is Risk?



- If we're going to find a portfolio or an allocation of assets to different stocks that minimizes risk, we have to pause for a moment and consider what is risk.
- Let's consider two stocks, XYZ, here that's gone up about ten percent, but as you can see, it's been very volatile.
- Here's the historical price of another stock, ABC. It also went up ten percent over this same time period, but as you can see, it was less volatile.
- So it's this volatility that we use as a measure of risk and that is simply the standard deviation of historical daily returns. Again, in this case, risk is measured as volatility or standard deviation of historical daily returns. That's the standard view of risk in most finance texts.
- There are other ways to view risk that we touch upon here and there through this course but the key is standard deviation of daily returns.

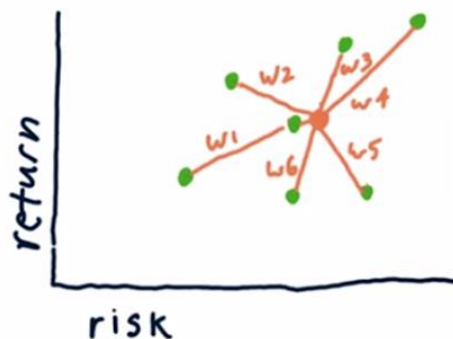
Visualizing Return vs Risk

Visualizing return vs risk



- Now, as you've probably heard over and over and over again, there's no reward without risk. And we often want to consider multiple stocks together and evaluate their risk versus return.
- So, one way we can do that is plot them on a scatter plot where each dot here represents one stock. So risk goes along this axis. Return is along this axis. So for instance, this stock here has higher risk, but about the same return as this stock. And then for comparison, this stock here has shown, historically about the same risk as this stock, but it's got a much higher return.
- Now we can build the portfolio by combining multiple assets, like the various stocks I've got plotted here and weighting each asset by a particular weight that represents its allocation within the portfolio.

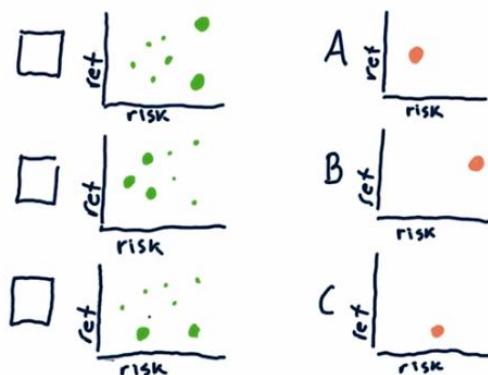
Visualizing return vs risk



- So each stock has a weight and when combined altogether we end up with a portfolio that has properties of each of the stocks. It's typically got a risk somewhere in the middle there and a return somewhere in the middle.

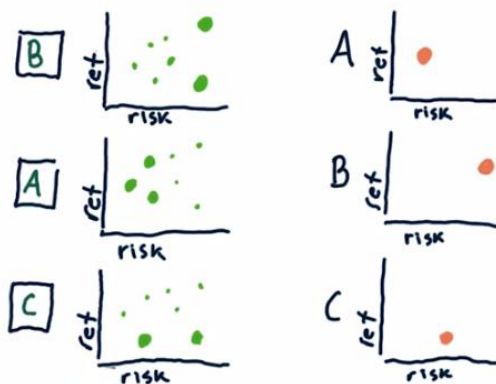
Building a Portfolio

Q: Building a portfolio



- I'd like you to consider these three portfolios. Now, each of the green dots represents an asset that's in the portfolio. And it's on this risk versus reward scatter plot. We've indicated the weights of the assets by the size of the dots. So for instance, this portfolio has most of its assets in these two equities, whereas this one has most of its assets in these three.
- Now, over here in this right column, I indicate with a big orange circle the risk/return level for several different portfolios, and what I want you to do is match these resulting portfolios to these weights. So, give it a go and I'll come back and show you the answer.

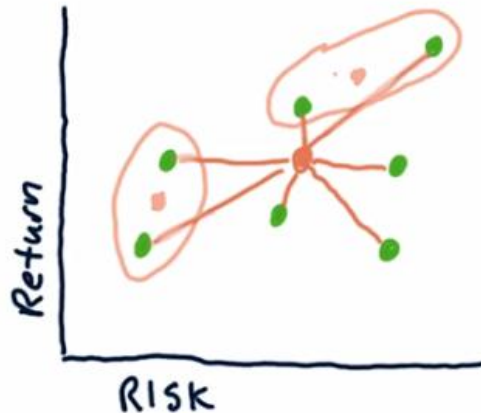
Q: Building a portfolio



- Okay. So in this portfolio, we're emphasizing risky assets and our portfolio ought to be somewhere around in here once we combine them. And of these three, this one seems to match the best so the answer here is B.
- If we look at this one, we're emphasizing less risky assets. So we should expect the portfolio to have a value at low risk. And of the three choices, A looks like the best match.
- This is the only one left, but note that it's essentially a combination, mostly of low return and low risk and high risk stocks. And so it's somewhere between and the answer there is C.

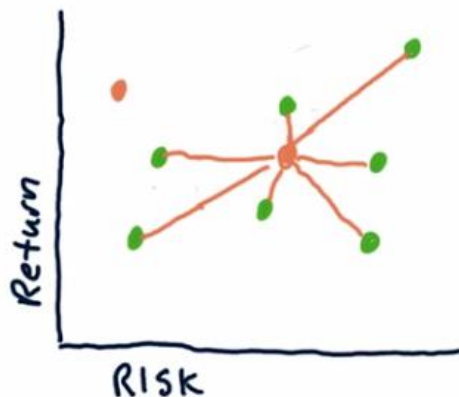
Can We Do Better?

Can we do better?



- So for quite a long time, people built portfolios this way. They would look at a bunch of assets, essentially equally weight them, and end up with a portfolio that behaved somewhat in between.
- Now, if you wanted a low-risk portfolio, you should focus on assets that are low risk out here. If you wanted, for instance, high return, you should focus on assets over here. In both those cases you would end up with a portfolio, say, here, or a portfolio, say, here.

Can we do better?



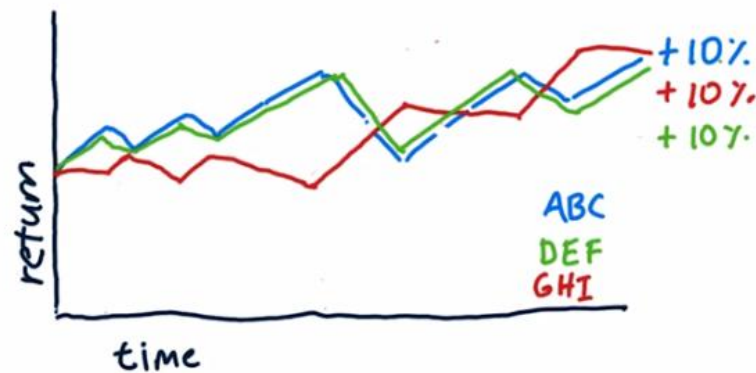
Harry Markowitz

- Now, can we do better? In other words, is it possible to get portfolios over here? So can we have a portfolio here that has a return similar to these assets over here, but risk similar to these assets over here?

- Indeed we can, and it's due to a man named Harry Markowitz, who won the Nobel Prize for his insight. What he discovered, and what people had been overlooking, was the relationship between stocks in terms of covariance.
- So the resulting performance of a portfolio, especially in terms of risk, is not just a factor, or a blend of the various risks, but it has to do with how they interact day to day. So indeed, if we pick the right stocks in the right proportions, we can get a portfolio that performs over here, that in fact, can have lower risk than any of the individual assets, and I'll show you how to do that in just a moment.
- Now before I do that, I want to mention though one other thing about Harry Markowitz. Up until the time of his discovery, most people viewed bonds as the lowest risk asset, in other words, if you wanted low risk, you should use bonds only.
- Markowitz showed that a blend of stocks and bonds is actually lower risk than either one of those by themselves. And I'll show you how that makes sense in a moment.

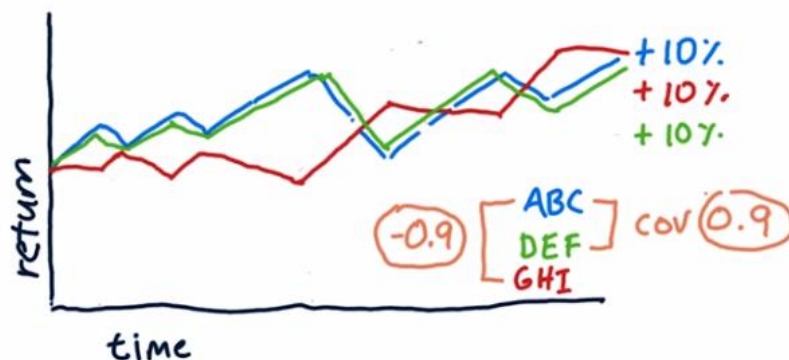
Why Covariance Matters

The importance of covariance



- So to illustrate the importance of covariance, we're going to take a look at a couple different stocks here. These are pretend, of course.
- So here's one, ABC, it's gone up 10%. Here's another stock, GHI, it's also gone up 10%. But notice it tends to zig when ABC zags, so it's not going in lockstep with ABC. Finally, here's our last stock that we'll take a look at, DEF. Note how it goes almost in lockstep with ABC.
- So, three stocks, two that move together very similarly, and one that moves opposite them sometimes, yet they've all provided about 10% return. What is the best portfolio we can build by combining these three different stocks?

The importance of covariance

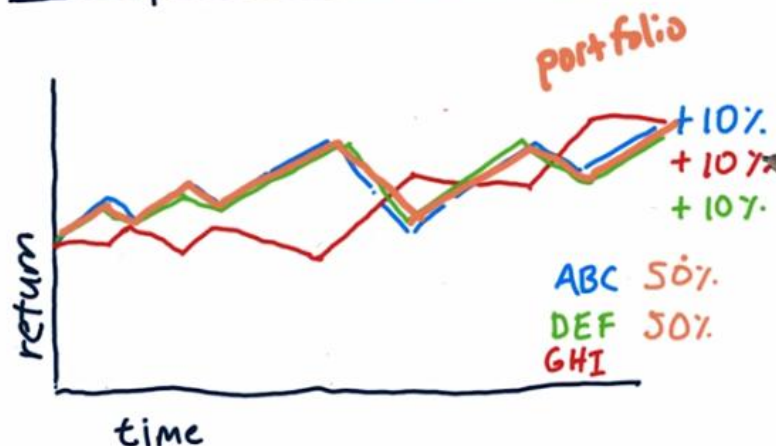


- So, let's consider their covariance, or how they move together, for a moment. So ABC and DEF move very similarly. And if we were to measure their covariance, and this is the covariance of daily returns, which if you're just looking at one stock versus another, that's the same as the

correlation coefficient of their daily returns. So 0.9 means that they move very closely to one another.

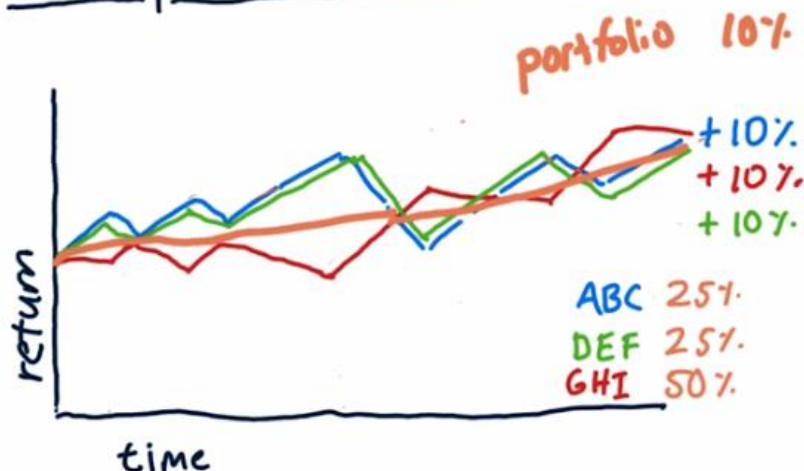
- Now, if we were to look at the co-variance of daily returns of ABC and GHI, we would find that they actually have a negative correlation. In other words, when ABC goes up, GHI tends to go down. So they actually have a negative correlation, or a negative covariance, whereas ABC and DEF have a positive covariance.
- Again now, what's the best way to blend these together?

The importance of covariance



- So let's consider a couple different portfolios, which are just weightings on these different assets. Let's look first at one that is 50% ABC and 50% DEF. So what's that one going to look like? Well, they both move very similarly, so this portfolio is going to track the two of them together. So the performance of this portfolio is going to look something like this.
- So that's fine, it ends up returning 10%, just like the two major assets that it holds, but there's no real advantage in blending these two assets because it just has the same volatility as each one of them by themselves.

The importance of covariance



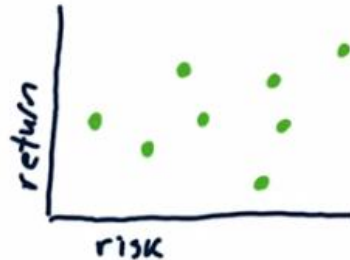
- So what if we try something a little bit different? What if we take 25% of ABC, 25% of DEF, so that's going to give us this same motion that we saw here when we combined those two, but, we put the rest of it in GHI?
- So now we're combining an anticorrelated asset with these other two. So what's this portfolio going to look like? Well, it's going to be a nice blend of them, with very low volatility, because when one zigs, the other zags. So this portfolio will also have a 10% return, but look how smooth it is. And that's because we're blending anticorrelated assets together.
- So we get the same return but lower volatility than any asset by itself. So all these assets by themselves had significant volatility, but when we put them together, we reduced volatility significantly. That's the magic of what Markowitz provided, and we'll look a little bit more at that in just a moment.

Mean Variance Optimization

Mean Variance Optimization (MVO)

Inputs

- Expected return
- Volatility
- covariance
- target return

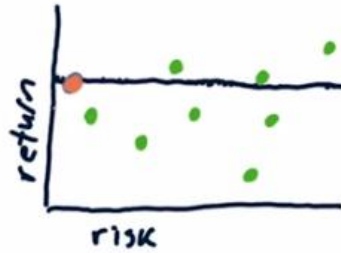


- What Markowitz added to the game was this consideration of variance and co-variance between individual stocks. And the recognition that you want to blend those together that have anti-correlation.
- So you can have a much lower risk portfolio if you combine assets that are anti-correlated. Or anti-varianced. [LAUGH]if that is a word. Because when one moves up, the other moves down. They cancel each other out. And you have much less volatility.
- Now of course you want, overall, all these assets to in general move up together. So often what we are looking for is anti-correlation in the short term and positive correlation in a longer term.
- But out of this work grew a number of algorithms, one of the key ones being mean variance optimization, which is a way of taking a potential set of assets and figuring out how they should be blended together by looking at their co-variance among other things.
- So let's consider this group of assets here and how we might combine them or allocate funds to them to provide a good portfolio.
- One thing I didn't point out before that I want to mention now. Is it generally the higher return stocks or assets, whatever they might be tend to also be the highest risk. So as we roll down the risk we tend to also look at lower return.
- Now again, how do we combine these into a good portfolio? Here's what goes in to a mean variance optimizer. These are the factors that it considers.
- So for each stock you have to provide an expected return. In other words, what do we think in the future it's going to provide in terms of return.
- You also provide volatility and covariance. Volatility is simply historically how volatile has each one of these assets been. Covariance is a matrix which shows between each asset and every other asset, what is the correlation of daily returns.
- So it takes all of these into account when it is searching for the right waiting for each of these stocks in a portfolio.
- So the last and perhaps one of the most important inputs to this is the target return. We can target a return anywhere from the max return asset to the min return asset. And then anything else between those, of course, we can accomplish by blending.

Mean Variance Optimization (MVO)

Inputs

- Expected return
- Volatility
- covariance
- target return



Output

- Asset weights for portfolio that minimize risk

- So let's suppose this is the level of our target return. So I'll just put a dotted line here so we can mark that for later. So the output of an optimizer is a set of weights, one weight for each asset, that provides the target return, but minimizes risk, so in the end we're looking for a portfolio that's out here.
- In other words it's got risk that's even lower than any of the individual assets. But it meets the target return. And this is possible because we consider covariance. We're only looking at individual risk we wouldn't be able to find the right blend that would bring us out here.
- Now that's it for optimization and the insights from Markowitz. But there's one more thing to look at with regards to optimizers that I'm going to show you next.

The Efficient Frontier



- There's one last topic I wanted to talk about with regard to portfolio optimizers and modern portfolio theory. Consider these assets, and also recognize that for any particular return level, there is an optimal portfolio.
- So let's suppose we picked this return level, and that this is the optimal portfolio. In other words, this portfolio reflects weightings of all these assets that provides the lowest risk for this particular return.
- Now, we can compute this for all the potential returns from the minimum return stock here up to the maximum. And if we did that for all of them, in fact there's an infinite number of them, so we couldn't really do it for all of them. You get a line.
- So our maximum return portfolio is up here and it would consist entirely of that one asset. But as we bring the return down we find a number of other portfolios all the way down to here.



- A couple of different things to observe that are interesting. One is as you reduce the return sometimes this curve comes back in this direction, indicating that actually the risk is increasing as we reduce the return.
- So for the most part, people don't want those sorts of portfolios. In other words, who wants a lower returning portfolio with more risk. So we typically look only at portfolios above this line.

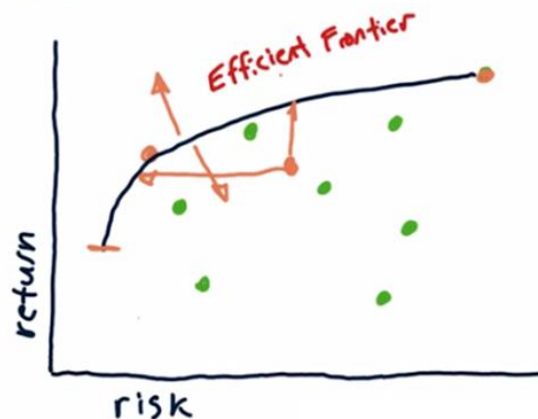
The Efficient Frontier



-As you might have guessed, the name of this line is the efficient frontier. So what does that mean, the efficient frontier?

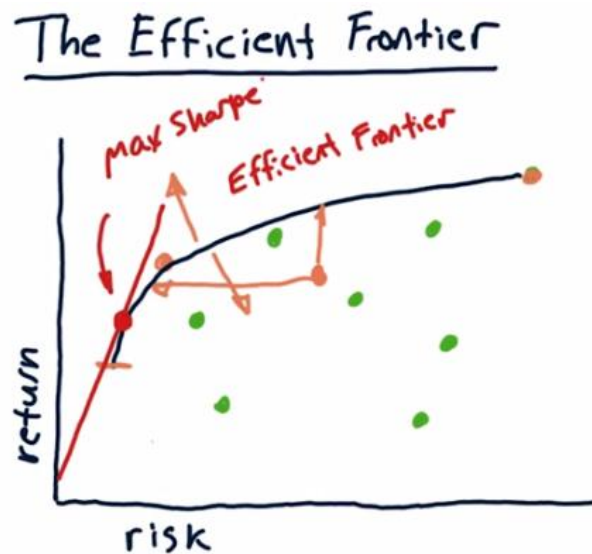
It means that there are essentially no portfolios out here. And that any portfolio that's on this side of the frontier, it's suboptimal in some way.

The Efficient Frontier



- So for instance, if you had a portfolio here, you are assuming this much risk, for a lower return than you could have, than if you were up here on the frontier. Or if you were targeting this much return, you could be over here with regard to risk.

- So any portfolio inside here is not efficient because it's either higher risk or lower return than it could be if we were on the efficient frontier.



- One last thing to mention about the efficient frontier is if you draw a tangent line from the origin to the frontier where it hits is the max sharp ratio portfolio for all of these assets.
- In practice the efficient frontier isn't used for that much other than as a theoretical device, but people do often like to plot the efficient frontier so they can see where their portfolio is in relation to the assets that they're using and where they could be in terms of efficiency.
- Okay, that is all. I will see you online soon. Thank you very much.