02-09 The Fundamental Law of Active Portfolio Management

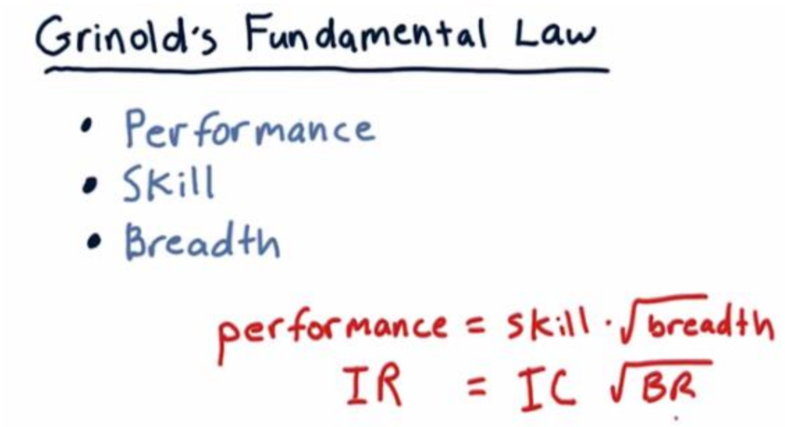
Compiled by Shipra De, Fall 2016

Overview

- You're probably familiar with Warren Buffett. He's one of the most successful investors ever. He's made a number of insightful comments over the years, many of which relate to this class.
- One of my favorites is, "only when the tide goes out do you discover who's been swimming naked." That's not the one for this lesson though.
- The one for this lesson is, "wide diversification is only necessary when investors do not know what they are doing." Mr.
- Buffett is talking about two things: investor skill and breadth, or the number of investments. This lesson is about how investment performance relates to those two factors.

(c) 2016 by Tucker Balch, all rights reserved.

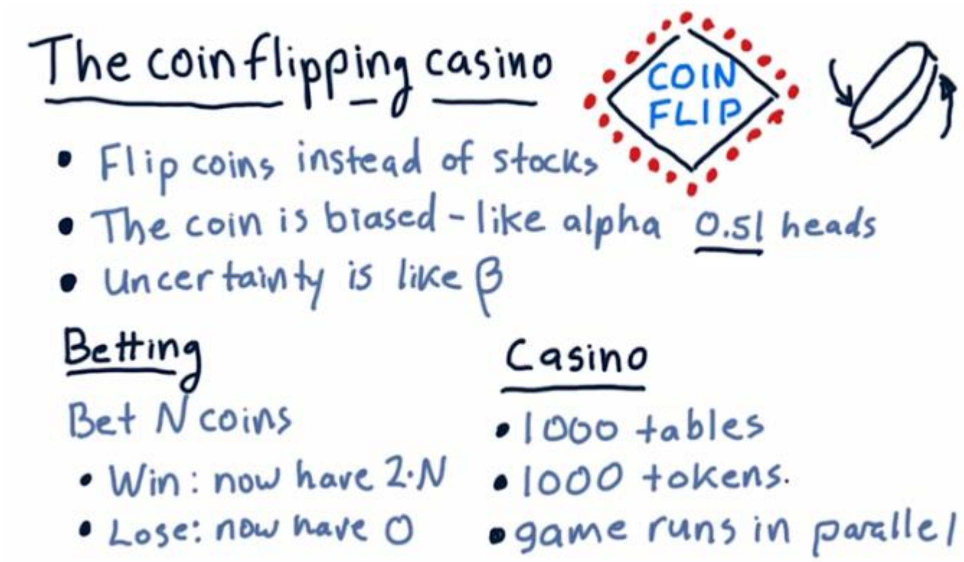
Grinold's Fundamental Law



- In the 1980s, Richard Grinold was seeking a method of relating performance, skill, and breadth in investing.
- So for instance, you might be a very skilled investor, meaning that you can pick stocks well, but you might not have many opportunities or breadth to exercise that skill.
- So Grinold felt that there must be some equation whereby we could combine skill and breadth to create an estimate of performance. He developed a relationship he called the Fundamental Law of Active Portfolio Management. I'm going to state it here at the beginning of the lesson, and then spend the rest of the lesson trying to justify it and show you why it makes sense.
- And here's his expression: performance is equal to skill times the square root of breadth. So we need some measure of skill and some measure of breadth. So if you want to improve your performance you can improve skill, or you can find more and more applications or methods or opportunities for applying that skill.
- So, as an example, breadth might relate to how many stocks you invest in and skill relates to your skill in choosing them. So, again you can invest in either improving your skill or improving your breadth.
- So performance is summarized in something called information ratio, which is very much like the sharpe ratio that we've discussed before, but it refers to the sharpe ratio of excess returns—in other words, the manner in which the portfolio manager is exceeding the market's performance.
- Skill is summarized in something called Information coefficient and breadth is just how many trading opportunities we have.
- Okay. So, we'll expand more and more on this as we go through the lesson.

(c) 2016 by Tucker Balch, all rights reserved.

The Coin Flipping Casino



- Now we're going to explore these ideas in a thought experiment called the coin flipping casino. So, instead of buying and selling stocks, we're going to flip coins and we're going to be on the outcome of whether those coins come up heads or tails.
- So that's analogous to a single investment in a stock, or a single trade in a stock, where you buy it and hold it and either you make money or you lose money. Well, we're going to flip a coin and either make money or lose money.
- Now the coin is biased. In other words, we have information about this coin, and we know that it's more likely to come up heads than tails. In fact, that probability is about 0.51 that it'll come up heads. So that's our edge, that's like our alpha when investing in stocks. And the uncertainty of the outcome is like beta in stock investing.
- Here's how betting works. So you can bet on any particular coin flip, you can bet n coins. If you win, in other words it comes up heads, you now have 2n coins. If you lose, they take your money. So this is called an even money bet. So on each outcome you either make n coins or you lose n coins.
- Here's how the casino works. There are 1,000 betting tables each with their own biased coin, and you have 1,000 tokens. So you can chose to bet however you like. In other words, you can put ten tokens on each of a hundred tables, you could put a thousand tokens on one table, you could put one token on all the tables.
- Once you've distributed all your tokens, which represent your bets, the coins are all flipped at once. In other words the game runs in parallel, and for each bet that you made you either get your winnings or they take the chips that you bet.
- So now that we've set up this environment for us to experiment with, I want you to think a little bit about it and think about how would you allocate these tokens? Is it better to bet all your tokens on just one coin flip, or is it better to bet on multiple coin flips at once?

(c) 2016 by Tucker Balch, all rights reserved.

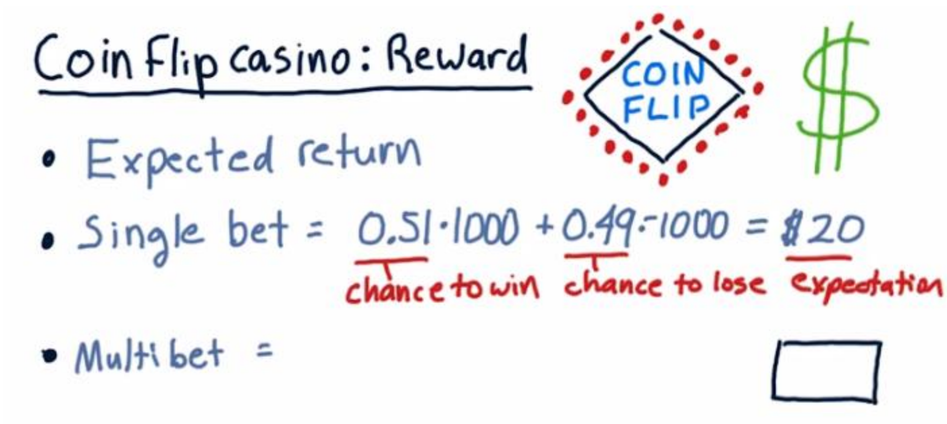
Which Bet is Better?

- So I want you to consider these two scenarios, two different ways to bet.
  - In one case, you put all of your 1000 tokens on a single table and So that's one way you might bet. And the outcome is going to depend on only what happens on the coin that's flipped on that single table.
  - Another approach is to put 1 token on each of the 1000 tables, and then all the coins are flipped in parallel and you get your reward or loss that way.
- Which one of these is better? Or is it the case that both of these are equivalent?

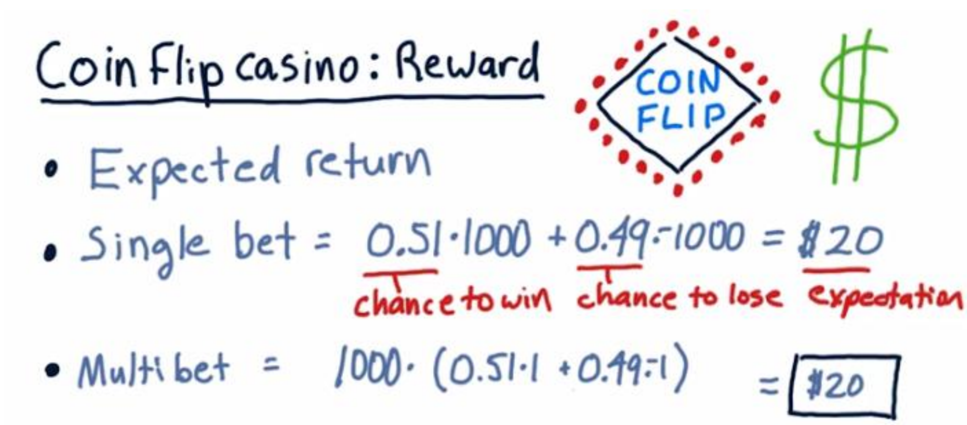
- Now, it would be fair for you to object and say, well, Professor Balch, what do you mean by better? I'll get onto that, but I think it's pretty clear, actually, that this is the best bet, and we'll spend some time explaining why.
- But the big picture is, this bet is very, very risky. There's a 49% chance you'll lose all your money with a single flip of the coin, whereas in this case, your coins are distributed over 1,000 individual small bets, and there's very little chance you'll lose all your money.
- And furthermore, the expected return for each of these bets is the same. It's just that this one has much lower risk.

(c) 2016 by Tucker Balch, all rights reserved.

Coin-Flip Casino: Reward?



- To figure out which of these scenarios is best, we need to consider reward and risk.
- Let's start here first with reward. So when we talk about reward in this situation, we're talking about our expected return. So in the single bet case, we bet 1000 chips at once.
- Our expected return is the chance that we'll win the bet times the return we get plus the chance that we would lose the bet times what we would lose.
- So, the chance that we're going to win is 0.51. Remember, it's a biased coin. And we have the opportunity to win 1000 tokens or $1000. The chance that we'll lose is 0.49 or 49%. And if we lose, we would lose a $1,000. So if you multiply this all out and add it up, you end up seeing that our expected return is $20.
- I'd like for you now to figure out what's our expected return or what's the reward in the multi-bet case where we bet 1000 individual chips on 1000 different tables. And all those coins are flipped at once. All biased coins with a 51% chance of winning for us. What's our expected return in that case?



- So, we make a bunch of individual bets, each one has the same chances as this one up here, except each one of those individual bets only provides a loss of $1 or a gain of $1. So, each of

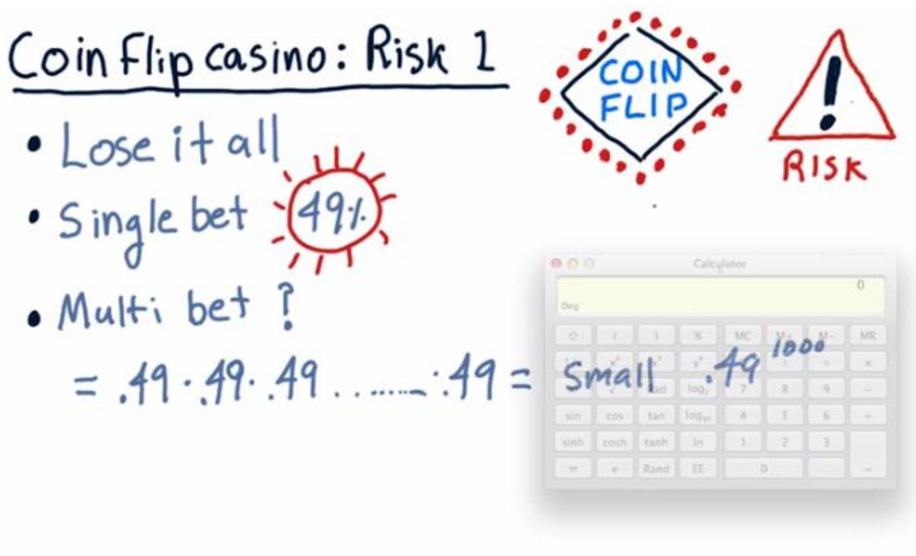
(c) 2016 by Tucker Balch, all rights reserved.

our individual bets has this 0.51 chance of winning $1, and 0.49 chance of losing $1. And we just have a thousand of those.

- So if you work out the numbers it turns out that this works out to be 0.02, and we multiply it by a thousand and we get $20.
- So what's remarkable, or at least interesting, is that for both of these ways of betting, our expected return is exactly the same. So the reward side of this determining which is better, is showing us that it's the same, so why should I say that this multibet approach is better? Well, it turns out that that's all about the risk, so let's look at the risk now.

(c) 2016 by Tucker Balch, all rights reserved.

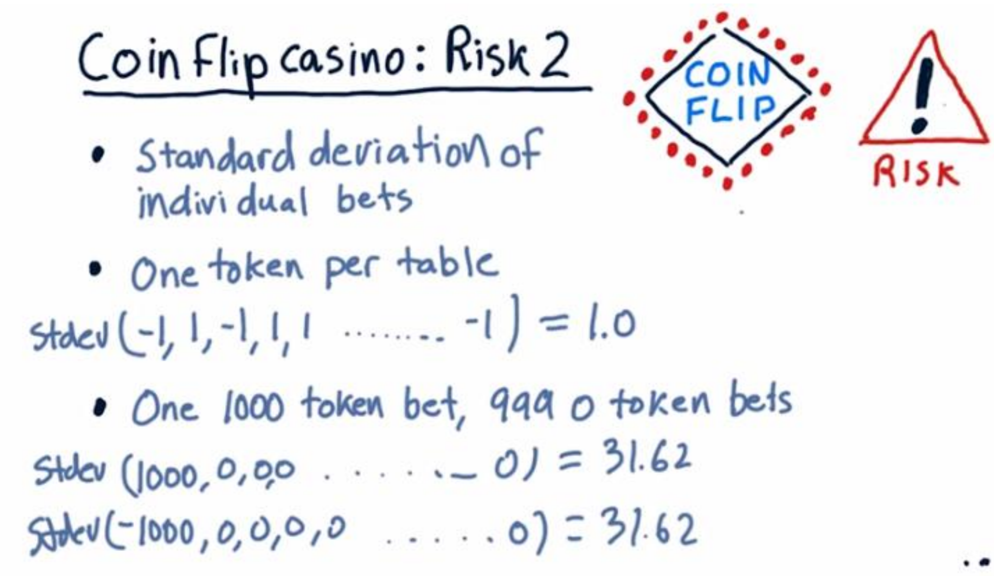
Coin-Flip Casino: Risk 1



- There's a number of ways that we might consider risk in this scenario. Let's look at first, this idea of what's the chance you might lose it all.
- So, you put out your bets on the thousands tables. The coins are flipped, in that event, what's the chance you'll lose all, all of your money? In the single bet case where you put all of your money on a single table, and the outcome is determined by the flip of a single coin, remember, even though it is biased in your favor, there still is a 49% change it'll come out against you.
- So if you put all of your money on a single table for one coin, there's a 49% chance that you'll lose all of it. That's pretty high. I mean consider that in the context of would you put your savings account on a single bet like that?
- Now, I want you to think a little bit, consider the multi-bet case where we put one token on each of the 1,000 tables. So there's 1,000 coin flips and our return is determined by the result of all those individual thousand coin flips. What is the chance that we would lose it all in that case? Well the answer to that is that it's the probability that we lose on the first table, times the probability that we lose on the second table, times the probability we lose on the third.
- And as you can see, we do this for all of the thousandth table and multiple those probabilities altogether, and that's the probability we would lose everything in the multiple bit scenario. That is a very, very small number. It's point four nine to the one thousandth power.
- How small is it? Let's try and calculate it on a calculator. So it's 0.49 raised to the 1,000th power. Let's see how small that is. It's so small it's not even a number. That's how small it is. Okay, let's look at another way to evaluate risk.

(c) 2016 by Tucker Balch, all rights reserved.

Coin-Flip Casino: Risk 2



- Another way that we can look at risk in this thought experiment is to consider the standard deviation of all those individual bets. So again consider we have a thousand different betting tables out there, and we have the opportunity to allocate our bets however we like.
- We could put 200 tokens on one table, 5 on another, and so on. We're looking at the two extreme cases here of course, but in any case, what is the standard deviation of the results of all those individual bets?
- So we're going to look at standard deviation after the fact, in other words, we're going to assume we bet already and we're looking at the outcome. So if we have one token per table we might have a loss of one token on one table, a gain on another, another loss, and a gain, and a gain and so on, all the way across our 1,000 tables.
- Now, we don't actually have to spend much time doing the math here. It turns out that the standard deviation for this case is pretty easy to calculate. Because for each table, the result is either -1 or 1. And it turns out that the standard deviation in this case is easily calculated as 1.0. So it doesn't matter what the particular outcome turns out to be. We know we're either going to lose 1 or gain 1 at every single table. And so our expected standard deviation there is 1.
- The outcome for the second scenario where we make one bet with a 1000 tokens and then essentially 0 bets on the other tables, turns out a little bit differently. So on this first table where we bet 1000 tokens, we either win 1000 or we lose 1000. And on all the other tables, the outcome is exactly 0. Same for this case where we lost on that first one.
- So again, we bet on one table 1,000 tokens, and we bet 0 tokens on the other 999. Reasoning it out this way enables us to calculate the standard deviation. Whichever way it goes whether we win or lose, the standard deviation on this event is the same. And the answer is 31.62.
- Key point here is, the standard deviation or risk is much, much larger. Well as you can see about 31 time larger, when we make that single bet on one table and no bets on the other tables.

(c) 2016 by Tucker Balch, all rights reserved.

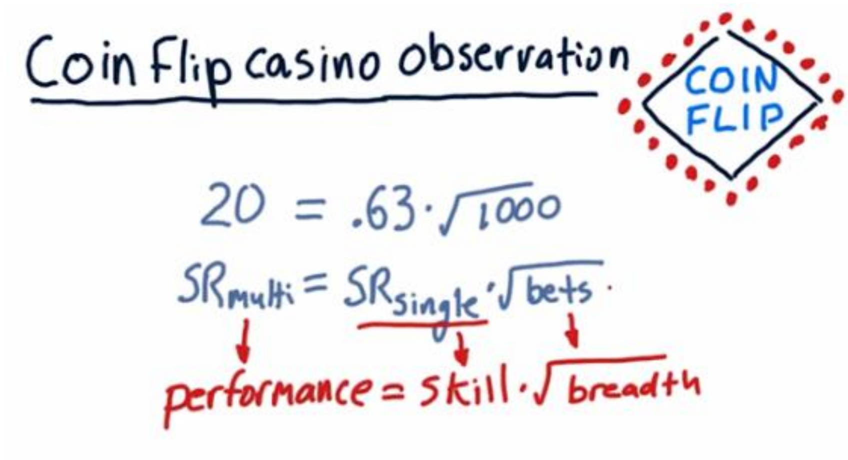
Coin-Flip Casino: Reward/Risk

- Now we're going to bring all these components together and consider risk adjusted reward and this is just like the sharp ratio that we've looked at in other contexts. In the single bet case we can calculate this ratio easily like this.
- So remember our reward or our expected return was $20 and our risk was $31.62. So if we divide this out, we get our sharp ratio. Or just the adjusted risk/reward ratio for this scenario. And it turns out to be 0.63.
- I'd like now for you to fill in the numbers for the multi bet case. In other words, a case where we point one token on each of the 1,000 tables and then get our return from that single event.

- So the reward or the return is the same. But remember the standard deviation of the risk was very much smaller. So we end up with a much larger ratio in the multi bet case.
- So the take home message here is. Even though we have the same expected return, we can have much lower risk and thus a much higher risk adjusted reward if we have many, many, bets.
- You know, a thousand bets instead of just a single bet.

(c) 2016 by Tucker Balch, all rights reserved.

Coin-Flip Casino: Observations



- So let's step back a moment and look at these results for these two extremes. So we have this one extreme where we bet all 1,000 chips on a single coin flip and another where we bet 1 chip on each of a 1,000 coin flips.
- Now the risk adjusted reward or the sharp ratio for the single that was 0.63, in other words if we bet all 1000 chips on a single flip of the coin our sharp ratio is 0.63. So let's call this SRsingle.
- Now the sharp ratio if we bet 1000 times in other words 1 chip on each of 1000 tables turns out to be 20. So what's the relationship between the two?
- Of course, 20 is about 40 times bigger than 0.63, but there's something more interesting there. And it turns out that it's this. So if we take the single bet case and multiply it by the square root of 1,000, we get the sharp ratio of our multi-bet scenario where we bet 1000 at once.
- So isn't that curious? So it turns out that in general, if you carry that scenario out to more examples, that if you split your bets evenly across multiple tables, this relationship will hold.
- In other words, the sharp ratio for the single bet, 1000 chips on one table, is our base case. And as we spread it out over more and more tables, the sharp ratio improves by the square root of that number of bets.
- And this relation is exactly like the fundamental law of active portfolio management. Namely, our overall performance is related to the skill, in the case of making a single pick, times the square root of the number of picks that we're able to make.
- So our coin flip casino has helped to show us how we can derive an equation like this for betting, which is what buying [LAUGH] stocks really is, and to show that you can improve things either by having more skill, (in case of coin flips, skill is how biased is the coin, in the case of investing, it's how good are you at predicting the future return of the stock) or you can improve your performance by increasing your breadth, but you only get to improve it by the square root of your breadth.

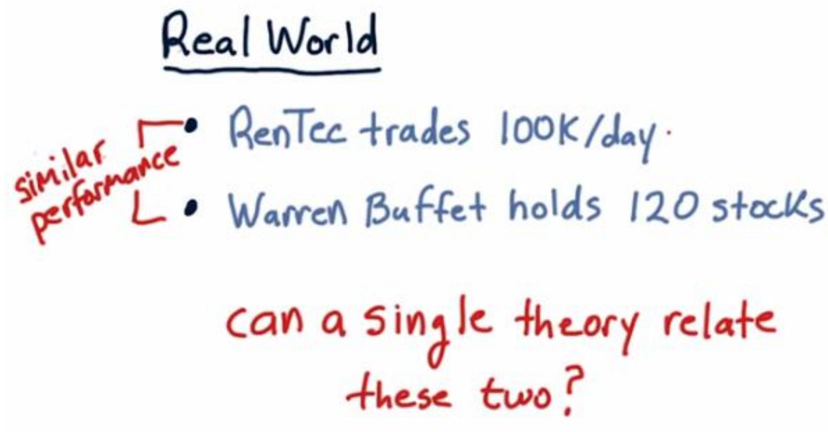
(c) 2016 by Tucker Balch, all rights reserved.

Coin-Flip Casino: Lessons

- Consider now the results of this thought experiment. Our casino enabled us to allocate our 1,000 tokens to 1,000 tables. We looked at two extreme cases. One where we put a small bet on all 1,000 tables, and another where we put all our money on one table.
- We saw that the expected return was the same in both cases, about $20, but the risk was substantially higher for the single bet case. This was true for both type of risk that we looked at, risk to lose everything, and risk as standard deviation.
- When we consider risk and reward together, we come away with three lessons.
    - One, higher alpha generates a higher sharpe ratio.
    - Two, more execution opportunities provides a higher sharpe ratio.
    - And three, sharpe ratio grows as the square root of breadth.

(c) 2016 by Tucker Balch, all rights reserved.

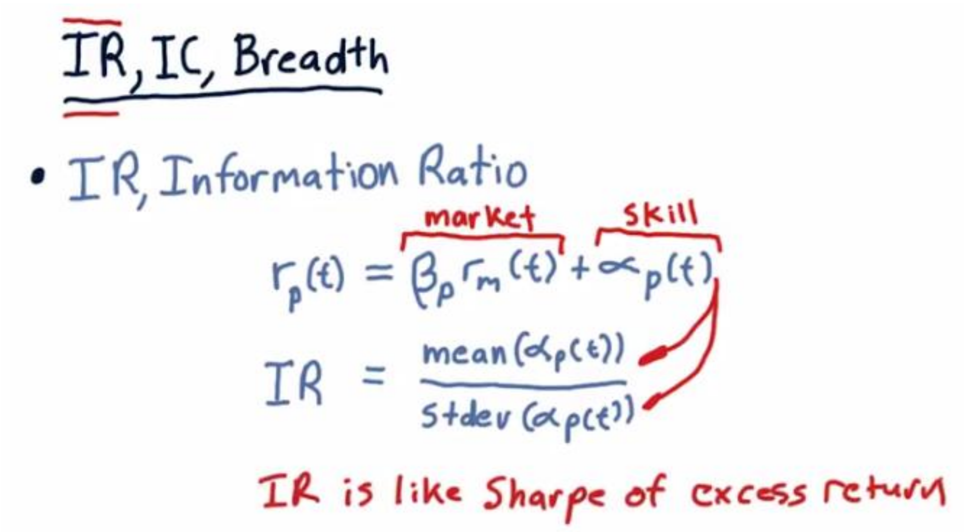
Back to the Real World



- Let's consider now the real world, and in particular hedge funds. So let's consider two real world funds. One is RenTec or Renaissance Technologies, it was founded by Jim Simons, a math and computer since professor, and he's had tremendous performance over the last several decades, and Warren Buffet, who runs Berkshire Hathaway.
- Both of these funds over the years have produced similar returns. But on the one hand, Warren Buffet holds maybe 120 stocks, and he doesn't trade much, he just holds them. Renaissance technology trades may be 100,000 times per day.
- Can a single theory relate these two? Can it account vastly widely differing numbers of trades per year, and can it relate the fact that they have similar performance and maybe different levels of skill? How does that all work together?
- Well, yes, there is a theory that can relate them, it's the fundamental law of active portfolio management, and I'm going to show it to you in just a moment.

(c) 2016 by Tucker Balch, all rights reserved.

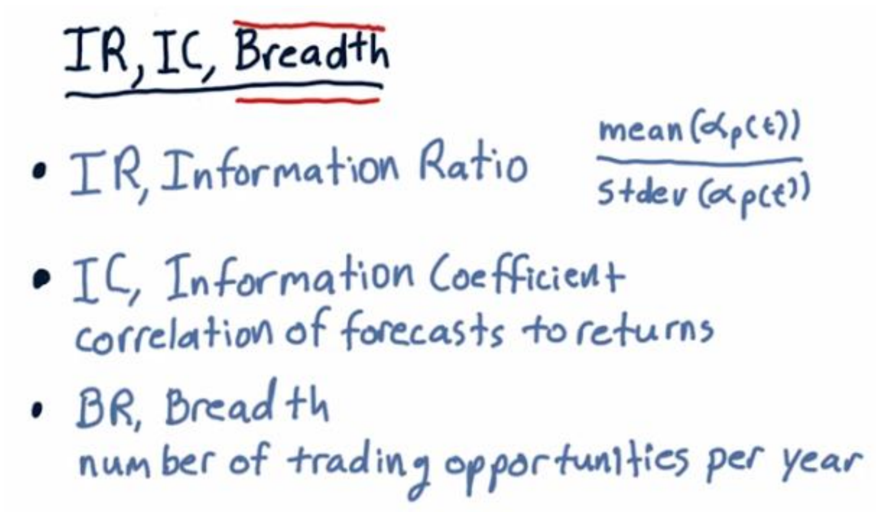
IR, IC, and Breadth



- Before I introduce the fundamental law, I've got to define a few terms. Let's start with information ratio. So recall, the return on our portfolio for a particular day is equal to the market component of the return, which is beta for that portfolio, times the return on the market for that day, plus this residual return.
- Another way to look at that is to say that this component of the return is due to the market, and this component is due to the skill of the fund manager. Remember, alpha is about skill.
- Now, we want to focus on the skill component. In other words, what is the fund manager bringing to the table? And we can calculate the Sharpe ratio, essentially, of this component, just like we can calculate the Sharpe ratio of an entire portfolio. But we can focus on this skill component, and essentially the Sharpe ratio of this skill component is the information ratio.
- It works like this. So the information ratio is the mean of all of the alpha components divided by the standard deviation of the alpha components. So this is our reward component. That's our risk component. And by the way, this is calculated by looking back historically at the daily values of alpha. And we take the mean and the standard deviation of them over time. You can find these by finding beta for the portfolio, calculating what the market return component was for each day, and then the difference is this residual or skill. So that's how you can compute historically the value of this information ratio.
- Now by the way, this information ratio applies in many different cases, not just in this fundamental law case. People use information ratio as a measure of manager performance all the time. It's fair to say that information ratio is essentially a Sharpe ratio of excess return, this part that's due to skill.

(c) 2016 by Tucker Balch, all rights reserved.

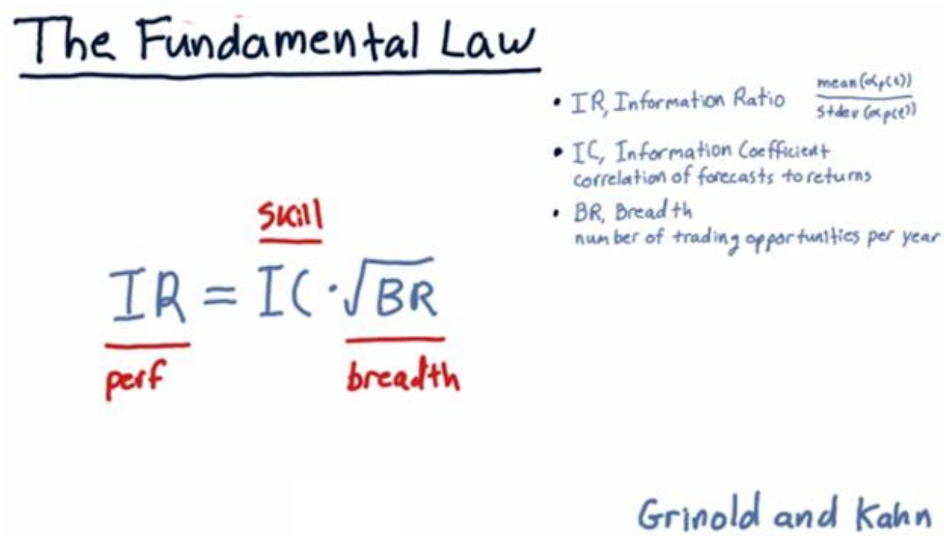
IR, IC, and Breadth (Cont.)



- IC, or the information coefficient, is just the correlation of the manager's forecast to actual returns. So for instance, if she made a forecast on IBM, that it would go up 1%, and it went up 0.5%, well, it's a positive correlation. This value can range from 0, where the correlation is not present, to 1, where it's very, very strong.
- Finally, BR, or breadth, represents the number of trading opportunities per year. So for example, if you're Warren Buffet, and you hold 120 stocks, and you just hold them all year, that's 120 trading opportunities. If, say, you're Jim Simons, and you trade 100,000 times per day, you multiply that times the number of trading days in a year, well, it turns out to be a very large number.
- Key thing is, even if it's just a portfolio where you buy and hold, you count the number of positions in that portfolio as the number of trading opportunities, because this is oriented around an annual measure.

(c) 2016 by Tucker Balch, all rights reserved.

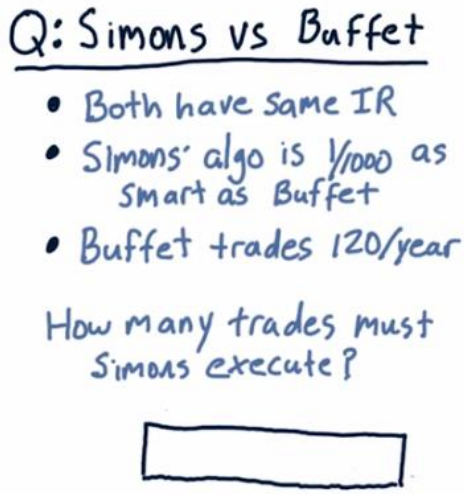
The Fundamental Law



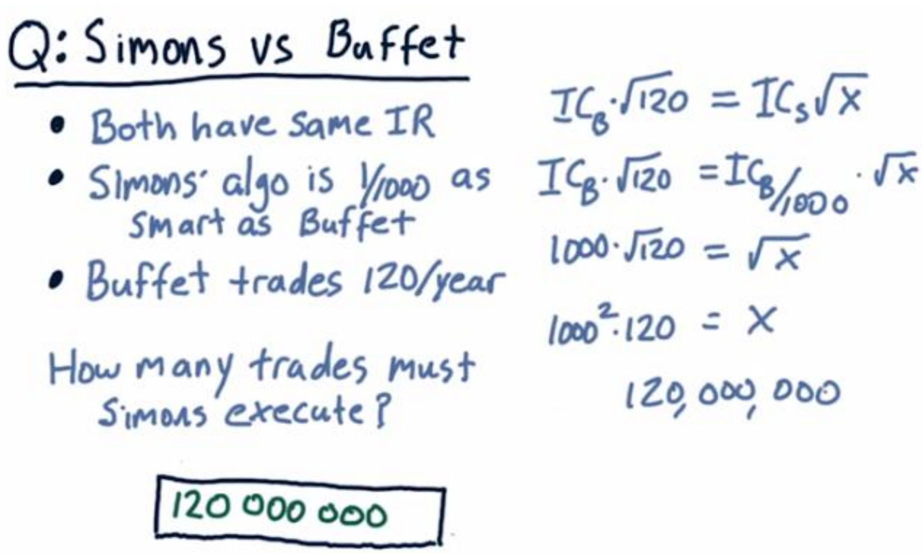
- Now that we've introduced these factors and to find them, we're now able to describe the fundamental law as expressed by Richard Grinold. And it's simple, it's just this. He was able to show mathematically that information ratio is equal to information coefficient times the square root of breadth.
- I'm just going to present it without proof here and encourage you to go look at Grinold's book if you're interested, for more detail. So, performance of the manager, or the fund, is due to the skill at making predictions times the square root of breadth.
- So that means, for instance, that yes, you may be very skilled but you've got to have opportunities to invest in order to operationalize your skill.
- Now if you want to improve your performance you can either focus on improving your skill or focus on improving your breadth.
- Sometimes it turns out, that it's a lot easier to increase breadth by finding additional stocks you might look at or additional markets. You might have some commodity trading strategy that works great. You can increase your breadth by looking at additional commodities.
- However, the strength of this increase, in other words, as you increase breadth, the overall performance only increases as the square root of that breadth. So it tapers off after some amount of time.
- It turns out, though, that it's very, very hard to improve your skill, so that is why, for instance, folks often focus on the breadth component of this equation.

(c) 2016 by Tucker Balch, all rights reserved.

Simons vs. Buffet



- Let's get back to that original question that motivated some of this, which is how can we relate the performance of Renaissance technologies, or Jim Simons, versus Warren Buffet? And the quiz here is for you to use the fundamental law to see if you can relate the two.
- Okay, so for this problem, assume that both Simons and Buffet have the same information ratio. I'm making that up, I don't know for sure that they do. But this is just a simplification for this problem.
- Assume that Simons' algorithms are only one 1000th as smart as Buffet. So that's another way of saying that their information coefficient is one 1000th the value of Buffet. Also assume that Buffet trades 120 times per year. Given that, how many trades must Simons execute in order to maintain the information ratio at the same level as Buffet?



(c) 2016 by Tucker Balch, all rights reserved.

- The answer is 120 million, so if Buffet trades only 120 times per year, Simons has to trade 120 million times.
- So here's how I arrived at that. We know that their information ratios are the same, so we just set the equation equal to each other. One for Simons and one for Buffet.
- So the information coefficient for Buffet, times square root of 120. This is Buffet's information ratio.
- Now this is Simon's information ratio. It's his information coefficient times the square root of x, which is what we're trying to find.
- Now first thing we can do is replace ICS with what we know is that it's one-one thousandth of Buffet's. Now we can multiply both sides by 1,000. And divide both sides by Buffet's IC. And we get this. And now we just square both sides and we're left with x or 120 million.
- Okay, that's it for the fundamental law of active portfolio management. I'll see you again soon, thanks.

(c) 2016 by Tucker Balch, all rights reserved.